POLARIMETRIC SCATTERING AND EMISSION PROPERTIES OF TARGETS WITH REFLECTION SYMMETRY

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Abstract

This paper investigates the symmetry of polarimetric scattering and emission coefficients of media with reflection symmetry. A reflection operator is defined and is used to create the images of electromagnetic fields and sources. The image fields satisfy Maxwell's equations, meaning that Maxwell's equations are invariant under the described reflection operations. By applying the reflection operations to media with reflection symmetry, the symmetry properties Of the Stokes parameters, characterizing the polarization state of thermal emissions, are shown to agree with misting experimental data. The first two Stokes parameters are symmetric with respect to the reflection plane, while the third and fourth Stokes parameters have odd symmetry. in active remote sensing, the symmetry properties of the polarimetric scattering matrix elements of deterministic targets and the polarimetric covariance matrix elements of random media or distributed targets are examined. For deterministic targets, the cross-polarized responses arc odd functions with respect to the symmetry direction, whereas the co-polarized responses are even functions. For distributed targets or random media, it is found that the correlations of co- and cross-polarized responses are anti-symmetric with respect to the reflection plane, while the other covariance matrix elements are symmetric. Consequently, in the cases of backscatter, the co- and cross-polarized components are completely ull-correlated when the incidence direction is on the symmetry plane. The derived symmetry properties of polarimetric backscattering coefficients agree with the predictions of a tw'c)-scale surface scattering model and existing sea surface HH and VV backscatter data. Finally, the conditions for a general type of media, i.e., bianisotropic media to be reflection symmetric are examined.

1. Introduction

This paper discusses the symmetry properties of the polarization components of active scattered fields and passive thermal radiations from media with reflection symmetry in light of recent significant interests in polarimetric active and passive remote sensing of geophysical media, particularly, wind-roughened ocean surfaces, which are symmetric to the wind direction, In active remote sensing, the HH or VV backscatter from wind-induced sea surfaces has been known to be symmetric with respect to the wind direction [Wentz et al., 1984]. However, symmetries of the other polarimetric backscattering coefficients, characterizing the mutual correlation between the electric fields collected with two arbitrary antenna polarizations, have not yet been discussed. In passive remote sensing, the polarization states of thermal radiations are described by a Stoke.s vector with four parameters. The first two Stokes parameters of sea surface brightness temperatures were found to be symmetric with respect to the ocean wind direction [Etkin et al., 1991; Wentz, 1992; Yueh et al., 1994c], while the third Stokes parameter was an odd function for emissions from corrugated soil surfaces [Nghiem et al,, 1991], from periodic water surfaces [Johnson et al., 1 992; Yueh et al., 1994a], and from sea surfaces at normal incidence [Dzura et al., 1992] and at incidence angles of 30 to 50 degrees [Yueh et al., 1994c]. Additionally, those observed symmetry properties of Stokes parameters are consistent with the results generated by a theoretical emission model for random rough surfaces [Yueh et al., 1993]. Although the experiment al evidence described above had suggested the symmetries of polarimetric active scattering and passive emission coefficients, there was no rigorous explanation based on the Maxwell equations. Here, this paper shows that the symmetries of electromagnetic fields scattered or emitted from reflection symmetric media can be derived from the Maxwell equations.

Section 2 demonstrates that Maxwell's equations are invariant under a described reflection operator. Section 3 describes how the polarization vector components of electric fields are transformed under the reflection operation. Section 4 presents the symmetry properties of thermal radiations with respect to the reflection plane, while Section 5 presents the symmetry properties of polarimetric scattering coefficients. The conditions for media to be reflection symmetric are discussed in Section 6.

2. Reflection operations and the invariance of Maxwell's actuations

This section introduces a reflection operator, which is used to define a complementary problem with the fields corresponding to the images of the fields in the original problem. It is shown that the image fields satisfy Maxwell's equations to demonstrate the invariance of Maxwell's equations under the reflection operations.

Without loss of generality, throughout this paper, the x-z plane, unless otherwise mentioned, is chosen as the reflection plane, with respect to which the reflection operation is applied. The reflection operator $\mathcal{R}$ is defined as follows: When $\mathcal{R}$ is applied to a scalar field $a(x, y, z)$, it creates a reflection image of the quantity by the following relation:

$$\mathcal{R}\,(a(x, y, z)) = a(x, -y, z) \tag{1}$$

When applied to a vector field $\overline{A}$, it creates another vector field

$$\mathcal{R}\left(\overline{A}(x, y, z)\right) = \hat{x}A_x(x, -y, z) - \hat{y}A_y(x, -y, z) + \hat{z}A_z(x, -y, z) \tag{2}$$

Given the above definition, it is straightforward to show that for an arbitrary vector field $\overline{A}$, the following commutation relations hold true when $\mathcal{R}$ operates together with the divergence and curl operators:

$$\begin{gathered} \nabla \cdot \mathcal{R}\left(\overline{A}\right) = \text{ `R } (17.2) \\ \nabla \text{ x } \mathcal{R}\left(\overline{A}\right) \text{ :----7<. } \left(\nabla \times \overline{A}\right) \end{gathered} \tag{3}$$

Hence, $\mathcal{R}$ is commutable with the divergence operator, and does not change the divergence of the vector field. In contrast, an additional minus sign is observed when the reflection and curl operations are reversed. This is because the reflection operation causes a sign change to the vector component perpendicular to the reflection plane, thus changing the handedness of the vector field.

Using $\mathcal{R}$, we can define a complementary problem with the fields and sources related to the images of those in the original problem as illustrated in Figure 1. Specifically, the fields in the complementary problem, indicated by the superscript 'prime', are defined as follows:

$$
\begin{aligned}
\bar{E}'(x,y,z) &= \mathcal{R}\left(\bar{E}(x,y,z)\right) \\
\bar{D}'(x,y,z) &= \mathcal{R}\left(\bar{D}(x,y,z)\right) \\
\bar{J}'(x,y,z) &= \mathcal{R}\left(\bar{J}(x,y,z)\right) \\
\rho'(x,y,z) &= \mathcal{R}\left(\rho(x,y,z)\right)
\end{aligned}
\tag{4}
$$

and

$$
\begin{aligned}
\bar{H}'(x,y,z) &= -\mathcal{R}\left(\bar{H}(x,y,z)\right) \\
\bar{B}'(x, //,2) &= -7< \left(\bar{B}(x,y,z)\right)
\end{aligned}
\tag{5}
$$

Note that unlike the electric fields and sources, the images of "magnetic" quantities $\bar{H}$ and $\bar{B}$ carry an opposite sign to the fields created by the reflection operator. That is, the reflection plane actually resembles a perfect magnetic-conducting wall. In contrast to the above image fields, another set of fields can be created by simulating the reflection plane as a perfect electrical-cc)ll(lllctillg wall with the signs of all "electric" quantities reversed, while maintaining the signs of magnetic quantities. However, these two sets of definitions lead to the same conclusions on the symmetry properties of polarimetric thermal radiations (Section 4), polarimetric scattering coefficients (Section 5), and criteria for a medium to be reflection symmetric (Section 6). Thus, we discuss only the case that the reflection plane corresponds to a perfect magnetic conducting plane.

In the original problem, the electric field $\bar{E}$, magnetic field $\bar{H}$, electric displacement $\bar{D}$ and magnetic flux density $\bar{B}$ are related to the charge density ρ and current density $\bar{J}$ through the Maxwell equations:

$$
\begin{aligned}
\nabla \cdot \bar{D} &= \rho \\
\nabla \cdot \bar{B} &= 0 \\
\nabla \times \bar{E} &= -\frac{\partial \bar{B}}{\partial t} \\
\nabla \times \bar{H} &= \bar{J} + \frac{\partial \bar{D}}{\partial t}
\end{aligned}
\tag{G}
$$

Given the commutation relations of $\mathcal{R}$ with the divergence and curl operators, it can be shown that the fields and sources in the complementary problem satisfy Maxwell's equations. For the Gauss' law of electric displacement, applying the commutation relation of the reflection operator with the divergence operator leads to

$$\nabla \cdot \bar{D}' = \nabla \cdot \mathcal{R}\left(\bar{D}\right) = \mathcal{R}\left(\nabla \cdot \bar{D}\right) = \mathcal{R}(\rho) = \rho' \tag{7}$$

For the Faraday induction law, which involves the curl operator, making use of the commutation relation results in

$$\nabla \times \bar{E}' = \nabla \times \mathcal{R}\left(\bar{E}\right) = -\mathcal{R}\left(\nabla \times \bar{E}\right) = -\frac{\partial \mathcal{R}\left(\bar{B}\right)}{\partial t} = -\frac{\partial \bar{B}'}{\partial t} \tag{8}$$

In a similar way, it can be shown that $\bar{B}'$ satisfies Gauss' law:

$$\nabla \cdot \bar{B}' = 0 \tag{9}$$

and the generalized Ampere's law holds for the image fields and currents:

$$\nabla \times \bar{H}' = \bar{J}' + \frac{\partial \bar{D}'}{\partial t} \tag{10}$$

Hence, the image fields and sources obtained through the reflection operations satisfy Maxwell's equations. In other words, Maxwell's equations are invariant under the reflection operations.

3. Transformation of the polarized components of electric fields

To facilitate the symmetry analyses of polarimetric scattering and emission coefficients, this section shows how the horizontally and vertically polarized components of electric fields are transformed under the reflection operation. In polarimetric remote sensing, an electric field in the far field regime propagating in the direction $\hat{r}$ ($= \hat{v} \times \hat{h}$) is decomposed into horizontal and vertical components, Figure 2,

$$\overline{E}(\theta, \phi) = E_h(\theta, \phi)\hat{h}(\theta, \phi) + E_v(\theta, \phi)\hat{v}(\theta, \phi) \tag{11}$$

where $\hat{h}$ and $\hat{v}$ represent the horizontal and vertical polarization vectors, respectively,

$$\begin{aligned} \hat{h}(\theta, \phi) &= \sin\phi\hat{x} - \cos\phi\hat{y} \\ \hat{v}(\theta, \phi) &= -\cos\theta\cos\phi\hat{x} - \cos\theta\sin\phi\hat{y} + \sin\theta\hat{z} \end{aligned} \tag{12}$$

Applying the reflection operator to the polarization vectors yields

$$\begin{aligned} \mathcal{R}\left(\hat{h}(\theta, \phi)\right) &= -\hat{h}(\theta, \phi) \\ \mathcal{R}\left(\hat{v}(\theta, \phi)\right) &= \hat{v}(\theta, \phi) \end{aligned} \tag{13}$$

Hence, the electric field $\overline{E}'$ in the complementary problem obtained from the reflection operation is

$$\overline{E}'(\theta, \phi) = \mathcal{R}\left(\overline{E}(\theta, \phi)\right) = -E_h(\theta, -\phi)\hat{h}(\theta, \phi) + E_v(\theta, -\phi)\hat{v}(\theta, \phi) \tag{14}$$

However, since the electric field $\overline{E}'$ can also be expressed in terms of the polarization vectors

$$\overline{E}'(\theta, \phi) = E'_h(\theta, \phi)\hat{h}(\theta, \phi) + E'_v(\theta, \phi)\hat{v}(\theta, \phi) \tag{15}$$

we obtain the following relations by comparing the above two equations

$$\begin{aligned} E'_h(\theta, -\phi) &= -E_h(\theta, \phi) \\ E'_v(\theta, -\phi) &= E_v(\theta, \phi) \end{aligned} \tag{16}$$

for any θ and ϕ. The above results are essential to deriving the symmetry properties of passive emission and active scattering coefficients in the next two sections.

4. Symmetry of **polarimetric brightness temperatures**

Thermal emissions from geophysical media are electromagnetic radiations from fluctuating currents representing the random thermal motions of charged particles with their energy states changed by the absorption and scattering effects of the media. If the media have geometric directional features or anisotropic medium properties, emission from such media becomes partially polarized with its polarization state described by a Stokes vector I_s with four parameters, T_v, T_h, U and V [Tsang et al., 1985]:

$$I_s = \begin{bmatrix} T_v \\ T_h \\ U \\ V \end{bmatrix} = c \begin{bmatrix} < E_v E_v^* > \\ < E_h E_h^* > \\ 2Re < E_v E_h^* > \\ 2Im < E_v E_h^* > \end{bmatrix} \tag{17}$$

where c is a constant and E_h and E_v are the horizontally and vertically polarized components of the radiated electric fields illustrated in Figure 2. The angular brackets denote the ('1ls(' 1111)1(' average, taking into consideration the random motion of charges. The asterisk denotes the complex conjugate.

For reflection symmetric deterministic targets, the complementary problem is the same as the original problem. in addition, because the thermal current $\bar{J}$ is random, having a zero mean and a second moment proportional to tile imaginary parts of permittivity and permeability, characterizing tile lossy effects of the medium according to the fluctuation and dissipation theorem [Landau and Lifshitz, 1958; Yueh and Kwok, 1992?], the random current is characterized by a symmetric random process (see Section 6). Hence, for each realization of the random current $\bar{J}(\bar{r})$, there is always a realization same as the reflection image of T](T).

For each realization of the random currents, Eq. (16) implies that the electric field $\overline{E}'$ radiated from the image current sources in the complementary problem must be related to the electric field radiated from the original random current sources by

$$
\begin{aligned}
&\left|E'_h(\theta, -\phi)\right|^2 = |E_h(\theta, \phi)|^2 \\
&\left|E'_v(\theta, -\phi)\right|^2 = |E_v(\theta, \phi)|^2 \\
&E'_v(\theta, -\phi)\, {E'_h}^*(\theta, -\phi) = -E_v(\theta, \phi) E^*_h(\theta, \phi)
\end{aligned}
\tag{18}
$$

For media with reflection symmetry, which allows us to drop the superscript, ensemble averaging the above equations over all realizations of random fluctuating currents leads to

$$
\begin{aligned}
&<|E_h(\theta, -\phi)|^2> = <|E_h(\theta, \phi)|^2> \\
&<|E_v(\theta, -\phi)|^2> = <|E_v(\theta, \phi)|^2> \\
&<E_v(\theta, -\phi)\, {E_h}^*(\theta, -\phi)> = -<E_v(\theta, \phi) E^*_h(\theta, \phi)>
\end{aligned}
\tag{19}
$$

Using Eq. (17), the above equations can be further translated into the symmetries of Stokes parameters:

$$
\begin{aligned}
T_v(\theta, \phi) &= T_v(\theta, -\phi) \\
T_h(\theta, \phi) &= T_h(\theta, -\phi) \\
U(\theta, \phi) &= -U(\theta, -\phi) \\
V(\theta, \phi) &= -V(\theta, -\phi)
\end{aligned}
\tag{20}
$$

Hence, T_v and T_h of reflection symmetric media are even functions of ϕ, while the third and fourth Stokes parameters are odd functions.

If a medium is reflection symmetric with respect to both x-z and y-z planes, applying

Stokes parameters T_v and T_h are independent of azimuth angle ϕ, and U and V are zeros, by successively applying the symmetry relations defined by Eq. (20) to any vertical plane.

Note that the symmetry relations described above also hold for random media, as long as the random distributions of permittivity and permeability functions are reflection symmetric. 'This is because for any realization of the medium with $\epsilon = \epsilon(x, y, z)$ and $\mu = \mu(x, y, z)$, there is always another realization with $\epsilon = \epsilon(x, -y, z)$ and $\mu = \mu(x, -y, z)$. (SW the discussions in Section 6). After averaging the Stokes parameters over all realizations of ϵ and μ, the Stokes parameters will satisfy the symmetry relations shown in Eq. (20).

The symmetry relations described in Eq. (20) are evident in Figure 3 illustrating the polarimetric brightness temperatures of sea surfaces measured with an incidence angle of 4(I degrees at K-band (19.35 GHz) by Yueh et al. [1994 c]. As shown, T_v, 7}, , and Q($=T_v - T_h$) are symmetric with respect to the wind direction ($\phi =$ 00), while the third Stokes parameter U is an odd function of ϕ. For easy comparison, second-order cosine (sine) series fits of T_v, 7'},, and Q (U) are also included. Note that because of the up/downwind asymmetric sea surface features, like foams and hydrodynamically modulated capillary waves, the Stokes parameters, are less symmetric with respect to the crosswind direction $\phi = 90°$.

5. Symmetry of polarimetric scattering coefficients

Recent interest in the symmetry of polarimetric scattering coefficients arouse from the predictions of theoretical scattering models [Borgeaud et al., 1987] and experimental observations for the polarimetric remote sensing of geophysical media [Nghiem et al., 1993], indicating that the co- and cross-polarized components of backscattered fields are

(111-coil'[,latc:(l for media with azimuthal symmetry, i.e., the media are reflection symmetric with respect to any plane perpendicular to the x-y plane. In particular, this relation has been successfully used to estimate the cross-polarization couplings existing in polarimetric radars using the responses from distributed targets with azimuthal symmetry [van Zyl, 19!)0]. Explanation of this complete de-correlation between co- and cross- polarized responses from azimuthally symmetric media was conducted by Nghiem et al. [1992] based on the condition givn by Eq. (1 1) in their paper. In contrast, this paper directly derives from the Maxwell equations the symmetry relations of polarimetric bistatic and monostatic scattering coefficients of reflection symmetric media, and Snows that the complete de-correlation of co-polarized and cross- polarized respon ses from targets with azimuthal symmetry is a special case of the general results.

Figure 4 depicts the scattering configuration, in which a target is illuminated by a plane wave with horizontally and vertically polarized components E_{hi} and E_{vi} incident from the direction θ_i and ϕ_i, and scattered into the direction θ and ϕ with the horizontally and vertically polarized electric field components denoted by E_{hs} and E_{vs}. The polarimetric scattering matrix elements $f_{\alpha\beta}$ with α and β being either h or v relate the incident electric fields to the scattered electric fields by the following equation:

$$\begin{bmatrix} E_{hs}(\theta,\phi) \\ E_{vs}(\theta,\phi) \end{bmatrix} = \frac{e^{ikr}}{r} \begin{bmatrix} f_{hh}(\theta,\phi;\theta_i,\phi_i) & f_{hv}(\theta,\phi;\theta_i,\phi_i) \\ f_{vh}(\theta,\phi;\theta_i,\phi_i) & f_{vv}(\theta,\phi;\theta_i,\phi_i) \end{bmatrix} \begin{bmatrix} E_{hi}(\theta_i,\phi_i) \\ E_{vi}(\theta_i,\phi_i) \end{bmatrix} \tag{22}$$

where r is the range from the target to the receiver. Similar notations are used with the addition of a superscript "prime" indicating the electromagnetic fields in the complementary problem. For example, the scattering matrix elements in the complementary problem are denoted by $f'_{\alpha\beta}$.

According to Eq. (1 6), the electric fields in the original and complementary problems are related by

$$
\begin{aligned}
E'_{hs}(\theta, -\phi) &= -E_{hs}(\theta, \phi) \\
E'_{vs}(\theta, -\phi) &= E_{vs}(\theta, \phi)
\end{aligned}
\tag{23}
$$

and

$$
\begin{aligned}
E'_{hi}(\theta_i, -\phi_i) &= -E_{hi}(\theta_i, \phi_i) \\
E'_{vi}(\theta_i, -\phi_i) &= E_{vi}(\theta_i, \phi_i)
\end{aligned}
\tag{24}
$$

Because the above equations are valid for arbitrary excitations, the scattering matrix elements of the original and complementary media are therefore related by

$$
\begin{aligned}
f'_{hh}(\theta, -\phi; \theta_i, -\phi_i) &= f_{hh}(\theta, \phi; \theta_i, \phi_i) \\
f'_{hv}(\theta, -\phi; \theta_i, -\phi_i) &= -f_{hv}(\theta, \phi; \theta_i, \phi_i) \\
f'_{vh}(\theta, -\phi; \theta_i, -\phi_i) &= -f_{vh}(\theta, \phi; \theta_i, \phi_i) \\
f'_{vv}(\theta, -\phi; \theta_i, -\phi_i) &= f_{vv}(\theta, \phi; \theta_i, \phi_i)
\end{aligned}
\tag{25}
$$

For deterministic targets with reflection symmetry, the complementary problem becomes the original problem. The superscript can be dropped, leading to the results that co-polarized scattering matrix elements are even functions with respect to x-z plane, while the cross- polarized components are odd functions. An interesting special case is that when $\phi = \phi_i = 0$, i.e., the incident and observation directions are on the reflection plane, the cross-polarized components f_{hv} and f_{vh} become zero, as expected.

For randomly distributed targets or rough surfaces, their scattering properties are described by the polarimetric bistatic scattering coefficients:

$$
\gamma_{\alpha\beta\mu\nu}(\theta, \phi; \theta_i, \phi_i) = \frac{4\pi < f_{\alpha\beta}(\theta, \phi; \theta_i, \phi_i) f^*_{\mu\nu}(\theta, \phi; \theta_i, \phi_i) >}{A \cos \theta_i} \tag{26}
$$

with α, β, μ and ν being either h or v. The factor "A" is the illuminated area used to normalize the scattering cross sections. The scattering coefficient $\gamma_{\alpha\beta\mu\nu}$ characterizes the correlation between $f_{\alpha\beta}$ and $f_{\mu\nu}$ with the ensemble average, denoted by <>, taken over the distributions of all the medium parameters, including the permittivity, permeability, and surface roughness.

For each realization of random medium or rough surface, the polarimetric bistatic scattering coefficients, denoted by $\gamma'_{\alpha\beta\mu\nu}$, of the complementary problem can be shown to be related to that of the original problem by

$$\gamma'_{\alpha\beta\mu\nu}(\theta,-\phi;\theta_i,-\phi_i)=\begin{cases}\gamma_{\alpha\beta\mu\nu}(\theta,\phi;\theta_i,\phi_i), & \alpha=\beta \text{ and } \mu=\nu, \text{ or } \alpha\neq\beta \text{ and } \mu\neq\nu;\\ -\gamma_{\alpha\beta\mu\nu}(\theta,\phi;\theta_i,\phi_i), & \text{Otherwise.}\end{cases} \tag{27}$$

by substituting the symmetry relations given in Eq. (25) into Eq. (26). If the random medium is reflection symmetric with respect to the x-z plane, the complementary problem reduces to the original problem, meaning that Eq. (27) reduces to

$$\gamma_{\alpha\beta\mu\nu}(\theta,-\phi;\theta_i,-\phi_i)=\begin{cases}\gamma_{\alpha\beta\mu\nu}(\theta,\phi;\theta_i,\phi_i), & \alpha=\beta \text{ and } \mu=\nu, \text{ or } \alpha\neq\beta \text{ and } \mu\neq\nu;\\ -\gamma_{\alpha\beta\mu\nu}(\theta,\phi;\theta_i,\phi_i), & \text{otherwise.}\end{cases} \tag{28}$$

Note that the scattering coefficients denoting the correlation between co- and cross-polarized responses have odd symmetry, while the others are symmetric.

In the case of backscattering, where transmitting and receiving antennas are collocated, the polarimetric backscattering coefficients are related to the bistatic scattering coefficients by

$$\sigma_{\alpha\beta\mu\nu}(\theta_i,\phi_i)=\cos\theta_i\,\gamma_{\alpha\beta\mu\nu}(\theta_i,\phi_i+\pi;\theta_i,\phi_i) \tag{29}$$

In the following discussions, $\sigma_{hhhh}, \sigma_{vvvv}, \sigma_{hvhv}$, and σ_{vhvh} are represented by conventional notations σ_{hh}, σ_{vv}, σ_{hv}, and σ_{vh}. Because the factor $\cos\theta_i$ is not a function of the azimuth angle ϕ_i, the symmetry of polarimetric backscattering coefficients can be easily reduced from that of bistatic scattering coefficients. Explicitly, the conventional backscattering coefficients and the correlations between two co-polarized or two cross-polarized responses are even functions of ϕ_i:

$$\begin{aligned}
\sigma_{hh}(\theta_i,-\phi_i) &= \sigma_{hh}(\theta_i,\phi_i) \\
\sigma_{vv}(\theta_i,-\phi_i) &= \sigma_{vv}(\theta_i,\phi_i) \\
\sigma_{hhvv}(\theta_i,-\phi_i) &= \sigma_{hhvv}(\theta_i,\phi_i) \\
\sigma_{hv}(\theta_i,-\phi_i) &= \sigma_{hv}(\theta_i,\phi_i) \\
\sigma_{vh}(\theta_i,-\phi_i) &= \sigma_{vh}(\theta_i,\phi_i) \\
\sigma_{hvvh}(\theta_i,-\phi_i) &= \sigma_{hvvh}(\theta_i,\phi_i)
\end{aligned} \tag{30}$$

while the correlations between co- and cross-polarized backscatters are odd functions:

$$\begin{aligned}
\sigma_{hhhv}(\theta_i,-\phi_i) &= -\sigma_{hhhv}(\theta_i,\phi_i) \\
\sigma_{hhvh}(\theta_i,-\phi_i) &= -\sigma_{hhvh}(\theta_i,\phi_i) \\
\sigma_{hvvv}(\theta_i,-\phi_i) &= -\sigma_{hvvv}(\theta_i,\phi_i) \\
\sigma_{vhvv}(\theta_i,-\phi_i) &= -\sigma_{vhvv}(\theta_i,\phi_i)
\end{aligned} \tag{31}$$

For the special case that the incidence direction is on the reflection plane (ϕ_i = O), Eq. (31) implies that

$$\sigma_{hhhv}(\theta_i,0) = \sigma_{hhvh}(\theta_i,0) = \sigma_{hvvv}(\theta_i,0) = \sigma_{vhvv}(\theta_i,0) = 0 \tag{32}$$

meaning that the co- and cross-polarized responses are un-correlated. This has been observed in the results of many scattering models for geophysical media with reflection symmetry, such as the random medium model by Borgeaud et al. [1987], but has not proved to be exact until now.

Note that Eel. (30) shows that the backscattering coefficients σ_{hh} and σ_{vv} are even functions of the azimuth angle ϕ_i. This has been well known in the microwave backscattering coefficients of wind-generated sea surfaces, which are symmetric with respect to the wind direction. For example, the SASS geophysical model function [Wentz et al., 1984], empirically relating the ocean wind vectors to the microwave backscattering coefficient σ_0 (σ_{hh} or σ_{vv}) by the following harmonics expansion:

$$\sigma_0 = A_0 + A_1 \cos \phi_i + A_2 \cos 2\phi_i \tag{33}$$

which is an even function Of the azimuth angle ϕ_i. Figures 5(a) and (b) illustrates σ_{vv} and σ_{hh}, calculated using t he S.4SS geophysical 111 odel function, as a funct ion Of ϕ_i for the wind speed of 11.5 m/s. The same plots also include the backscatters measured by NUSCAT during Surface Wave Dynamics Experiment (SWADE) in 1991 [Nghiem et al., 1993b]. As shown, σ_{hh} and σ_{vv} are symmetric functions Of ϕ_i. To study the symmetry properties of the other polarimetric backscattering coefficients, also included in Figure 5 are t he theoretical polarimet ric backscat tering coefficients calculated using a t wo scale sea surface scattering model originally developed by Durden and Vesecky [1985] for σ_{vv} and σ_{hh} and generalized to polarimetric scattering by Yuel ı et al. [1993]. F 'igure 5(c) 1"(WYIIS a 180 degrees phase change 111 ρ_{hhhv} and ρ_{hvvv} at the upwind ($\phi_i = 0°$ degrees) and downwind (1800) direct ions, indicating that theoretical correlations between co- and cross-polarized responses from sea surfaces have odd symmetry - as proved in this paper based on reflection symmetry assumption. Note that as expected, all backscattering coefficients are less symmetric with respect to the crosswind direction because of the up/downwind asymmetric features in wind-induced sea surfaces.

6. Conditions for Reflection Symmetry

Though reflect ion symmetry has been used frequent ly to verify solutions or simplify problems, the criteria for media to be reflection symmetric have not yet been addressed. In fact this is a question that cannot be answered based on electromagnetics alone, because the medium parameters, describing the constitutive relations, such as the permittivity and permeability, are determined by how the particles react under the influence of CICC.-

tromagnetic forces together with other mechanical forces present in the media. Hence, the most general approach to decide whether a medium is reflection symmetric should be to show that all relevant physical laws, which govern particle motions, such as Lorentz force for charged particles and Schrödinger equation in quantum mechanics, have to be reflection symmetric. However, to avoid such complexity and to discuss cases as general as possible without being constrained to specific physical mechanisms or governing laws, this paper investigates a general type of media: bianisotropic media [Kong, 1986].

In electromagnetics, the macroscopic medium properties are described by a set of constitutive relations, which connect the vector fields $\overline{E}$, $\overline{H}$, $\overline{D}$, and $\overline{B}$ together. The constitutive relations of bianisotropic media are given by

$$\begin{aligned} \overline{D} &= \overline{\overline{\epsilon}} \cdot \overline{E} + \overline{\overline{\chi}} \cdot \overline{H} \\ \overline{B} &= \overline{\overline{\zeta}} \cdot \overline{E} + \overline{\overline{\mu}} \cdot \overline{H} \end{aligned} \tag{34}$$

where $\overline{\overline{\epsilon}}$ and $\overline{\overline{\mu}}$ are the permittivity and permeability tensors, $\overline{\overline{\chi}}$ describes how electric displacement reacts in the presence of magnetic field and $\overline{\overline{\zeta}}$ causes the magnetic flux density to respond to the electric field. Similarly, the constitutive relation of the medium in the complementary problem is described by

$$\begin{aligned} \overline{D}' &= \overline{\overline{\epsilon}}' \cdot \overline{E}' + \overline{\overline{\chi}}' \cdot \overline{H}' \\ \overline{B}' &= \overline{\overline{\zeta}}' \cdot \overline{E}' + \overline{\overline{\mu}}' \cdot \overline{H}' \end{aligned} \tag{35}$$

For convenience, the tensors described above shall be represented by a three-by-three matrix in the Cartesian coordinate. For example, the matrix $\overline{\overline{\epsilon}}$ is denoted in the Cartesian coordinate as

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} \tag{36}$$

Similar notations are used for all the other tensors.

Because the reflection operations defined by Eqs. (4) and (5) hold for arbitrary field quantities, it is straightforward to show that $\bar{\bar{\epsilon}}'$, $\bar{\bar{\mu}}'$, $\bar{\bar{\chi}}'$ and $\bar{\bar{\zeta}}'$, evaluated at the position i' = (J", y, z) are related to $\bar{\bar{\epsilon}}$, $\bar{\bar{\mu}}$, '\, and $\bar{\bar{\zeta}}$, evaluated at $\bar{r} = (x, -y, z)$ by

$$\begin{bmatrix} \epsilon'_{xx} & \epsilon'_{xy} & \epsilon'_{xz} \\ \epsilon'_{yx} & \epsilon'_{yy} & \epsilon'_{yz} \\ \epsilon'_{zx} & \epsilon'_{zy} & \epsilon'_{zz} \end{bmatrix}_{\bar{r}=(x,y,z)} = \begin{bmatrix} \epsilon_{xx} & -\epsilon_{xy} & \epsilon_{xz} \\ -\epsilon_{yx} & \epsilon_{yy} & -\epsilon_{yz} \\ \epsilon_{zx} & -\epsilon_{zy} & \epsilon_{zz} \end{bmatrix}_{\bar{r}=(x,-y,z)} \tag{37}$$

$$\begin{bmatrix} \mu'_{xx} & \mu'_{xy} & \mu'_{xz} \\ \mu'_{yx} & \mu'_{yy} & \mu'_{yz} \\ \mu'_{zx} & \mu'_{zy} & \mu'_{zz} \end{bmatrix}_{\bar{r}=(x,y,z)} = \begin{bmatrix} \mu_{xx} & -\mu_{xy} & \mu_{xz} \\ -\mu_{yx} & \mu_{yy} & -\mu_{yz} \\ \mu_{zx} & -\mu_{zy} & \mu_{zz} \end{bmatrix}_{\bar{r}=(x,-y,z)} \tag{38}$$

$$\begin{bmatrix} \chi'_{xx} & \chi'_{xy} & \chi'_{xz} \\ \chi'_{yx} & \chi'_{yy} & \chi'_{yz} \\ \chi'_{zx} & \chi'_{zy} & \chi'_{zz} \end{bmatrix}_{\bar{r}=(x,y,z)} = \begin{bmatrix} -\chi_{xx} & \chi_{xy} & -\chi_{xz} \\ \chi_{yx} & -\chi_{yy} & \chi_{yz} \\ -\chi_{zx} & \chi_{zy} & -\chi_{zz} \end{bmatrix}_{\bar{r}=(x,-y,z)} \tag{39}$$

and

$$\begin{bmatrix} \zeta'_{xx} & \zeta'_{xy} & \zeta'_{xz} \\ \zeta'_{yx} & \zeta'_{yy} & \zeta'_{yz} \\ \zeta'_{zx} & \zeta'_{zy} & \zeta'_{zz} \end{bmatrix}_{\bar{r}=(x,y,z)} = \begin{bmatrix} -\zeta_{xx} & \zeta_{xy} & -\zeta_{xz} \\ \zeta_{yx} & -\zeta_{yy} & \zeta_{yz} \\ -\zeta_{zx} & \zeta_{zy} & -\zeta_{zz} \end{bmatrix}_{\bar{r}=(x,-y,z)} \tag{40}$$

For llc)lllc)g('11('oils, bianisotropic media with reflection symmetry with respect to x-z plane, the above relations imply that

$$\begin{aligned} \epsilon_{xy} &= \epsilon_{yx} = \epsilon_{yz} = \epsilon_{zy} = 0 \\ \mu_{xy} &= \mu_{yx} = \mu_{yz} = \mu_{zy} = 0 \\ \chi_{xx} &= \chi_{yy} = \chi_{zz} = \chi_{xz} = \chi_{zx} = 0 \\ \zeta_{xx} &= \zeta_{yy} = \zeta_{zz} = \zeta_{xz} = \zeta_{zx} = 0 \end{aligned} \tag{41}$$

1 3ianisotropic media satisfying the above symmetry conditions have been used by Yang and Uslenghi [1993] in their analysis of planar bianisotropic waveguides.

Special cases of bianisotropic media include isotropic media, biaxial media, chiral media, and gyrotropic media. For isotropic media with $\bar{\bar{\chi}} = \bar{\bar{\zeta}} = 0$ and $\bar{\bar{\epsilon}}$ and $\bar{\bar{\mu}}$ being a scalar, Equations (37) and (38) reduce to

$$\begin{aligned} \epsilon'(x, -y, z) &= \epsilon(x, y, z) \\ \mu'(x, -y, z) &= \mu(x, y, z) \end{aligned} \tag{42}$$

Therefore, for deterministic, isotropic media the media are reflection symmetric if and only if ϵ and μ are even functions of y. in contrast, for random media with ϵ and μ described by spatial random processes, reflection symmetry means that the governing random processes for ϵ and μ are symmetric with respect to the reflection plane. In other words, if $\epsilon = \epsilon(x, y, z)$ and $\mu = \mu(x, y, z)$ are realizations of the random medium, then there exists another realization with $\epsilon = \epsilon(x, -y, z)$ and $\mu = \mu(x, -y, z)$.

For a homogeneous biaxial medium, $\overline{\overline{\chi}}$ and $\overline{\overline{\zeta}}$ are zero, the permeability is a scalar, and the permittivity is a diagonal tensor in the principal coordinate described by

$$\overline{\overline{\epsilon}} = \epsilon_x \hat{x}'\hat{x}' + \epsilon_y \hat{y}'\hat{y}' + \epsilon_z \hat{z}'\hat{z}' \tag{43}$$

If the medium is rotated with respect to the y-axis by an angle ψ, then the matrix representation of $\overline{\overline{\epsilon}}$ in the laboratory frame becomes

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon_x \cos^2\psi + \epsilon_z \sin^2\psi & 0 & (\epsilon_z - \epsilon_x)\cos\psi\sin\psi \\ 0 & \epsilon_y & 0 \\ (\epsilon_z - \epsilon_x)\cos\psi\sin\psi & 0 & \epsilon_x \sin^2\psi + \epsilon_z\cos^2\psi \end{bmatrix} \tag{44}$$

Hence, the medium is reflection symmetric with respect to the x-z plane, but not the other vertical planes.

Unlike biaxial media, a gyrotropic medium is anisotropic but non-reciprocal [Kong, 1987], including either electrically gyrotropic media or magnetically gyrotropic media. The general form of $\overline{\overline{\epsilon}}$ for an electrically gyrotropic medium, such as electron plasma under the influence of an applied external DC magnetic field, with the optical axis pointing in the z-direction is given by:

$$\overline{\overline{\epsilon}} = \begin{bmatrix} \epsilon & i\epsilon_g & 0 \\ -i\epsilon_g & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \tag{45}$$

As discussed, the medium is reflection symmetric with respect to the x-y plane, but not the x-z nor y-z planes.

For chiral media, which have been receiving significant attention in recent years, the constitution relation is

$$\overline{D} = \epsilon\overline{E} + i\chi\overline{H}$$
$$\overline{B} = -i\chi\overline{E} + \mu\overline{H} \tag{46}$$

Hence, according to Eq. (41), homogeneous chiral media are not reflection symmetric with respect to any vertical plane. However, the reflection image of a right-handed chiral medium ($\chi > 0$) is a left-handed chiral medium ($\chi < 0$), and vice versa,

7. Summary

This paper has analyzed the symmetry properties of polarimetric active scattering and passive emission coefficients of media with reflection symmetry. For passive remote sensing, it is shown that the first two Stokes parameters of thermal radiations from reflection symmetric objects are even functions with respect to the symmetry plane, while the third and fourth Stokes parameters are odd functions. For active scattering, the correlation coefficients of co- and cross-polarized responses are antisymmetric, unlike the other polarimetric scattering coefficients. These symmetry relations are shown to agree with the azimuthal signatures of microwave backscattering and emission measurements of sea surfaces and artificially constructed surfaces with directional features reported in the literature.

Potential applications of the studied symmetry relations include the detection of geophysical directional features using polarimetric remote sensing measurements. For example, the symmetry property of each polarimetric scattering or emission parameter derived in this paper suggests an appropriate functional form for the geophysical model function which relates the radar or radiometer observations to the direction of geophysical features. That is, while a cosine series of the azimuth angle was appropriate for the HH and VV backscattering coefficients in the SASS model function [Wentz et al., 1984], and the brightness temperatures at horizontal and vertical polarizations in the SSM/I model function [Wentz, 1 992] of sea surfaces, a sine series should be used in the geophysical model function of the other polarimetric scattering and emission coefficients which have odd symmetry [Yueh ('t al., 1994b, c].

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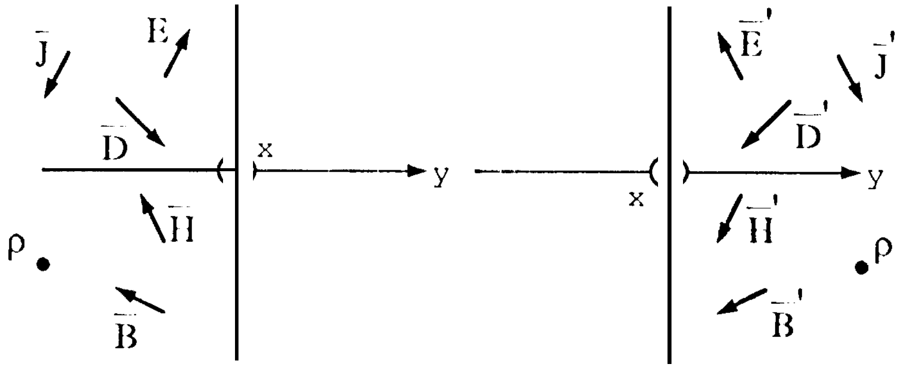
List of Figures



(a) Original. problem (b) Complementary problem

Figure 1

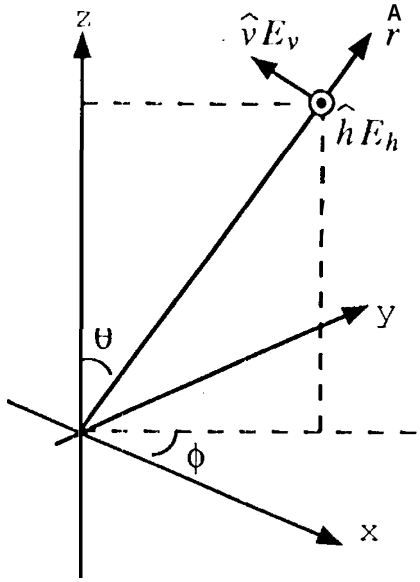
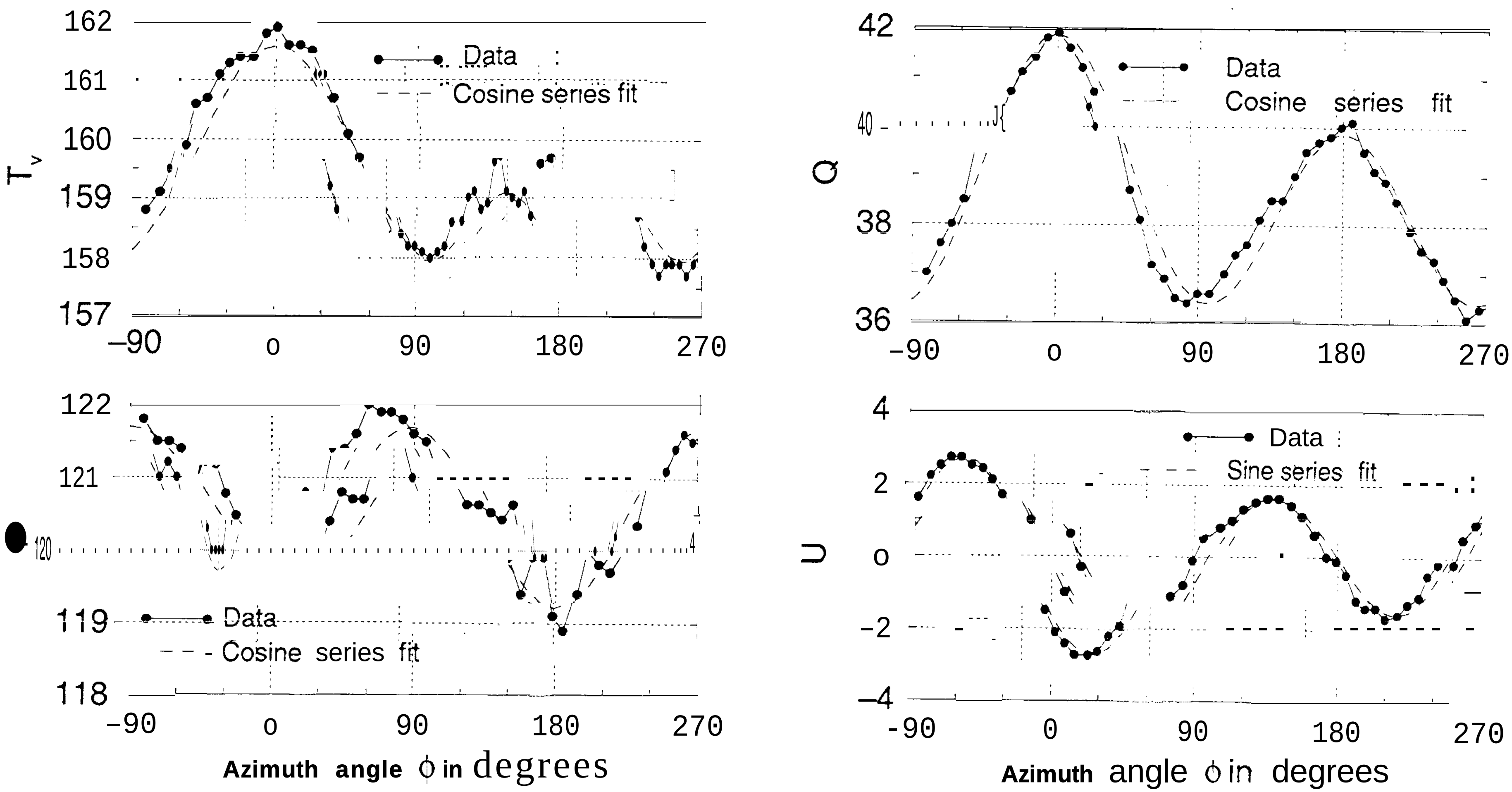


Figure 2



Data
Cosine series fit
T_v
162
161
160
159
158
157
-90
0
90
180
270
Q
42
40
38
36
122
121
120
119
118
U
4
2
0
-2
-4
Sine series fit
Azimuth angle ϕ in degrees

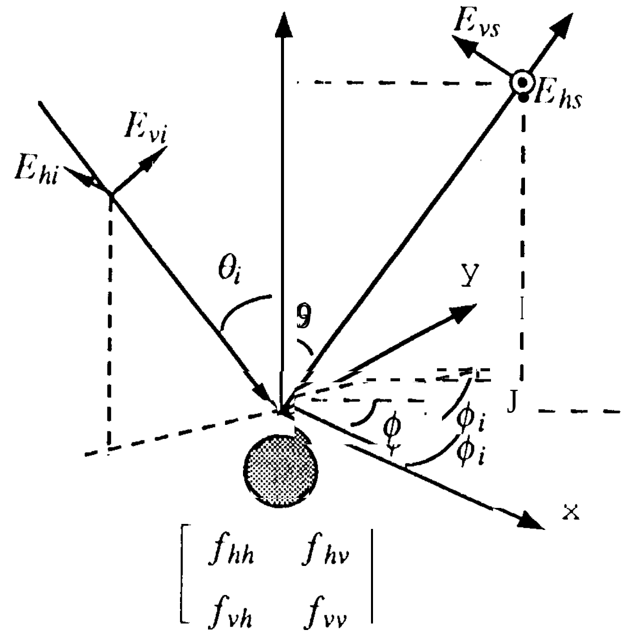
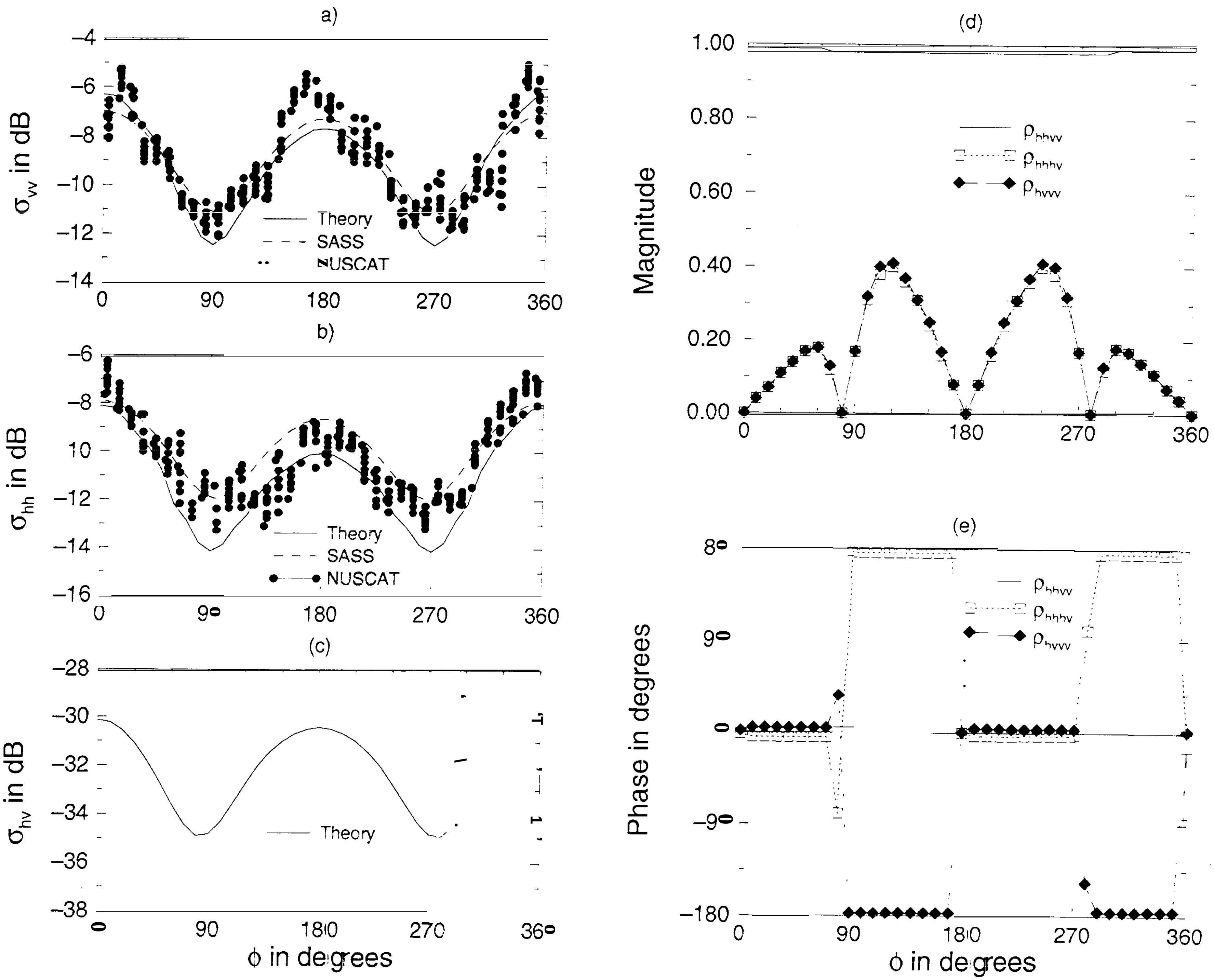


Figure 4



a)
σ_{vv} in dB
Theory
SASS
NUSCAT
b)
σ_{hh} in dB
Theory
SASS
NUSCAT
(c)
σ_{hv} in dB
Theory
ϕ in degrees
(d)
Magnitude
ρ_{hhvv}
ρ_{hhhv}
ρ_{hvvv}
(e)
Phase in degrees
ρ_{hhvv}
ρ_{hhhv}
ρ_{hvvv}
ϕ in degrees